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### Evaluating a satellite surveillance system

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## EVALUATING A SATELLITE SURVEILLANCE SYSTEM

ROLF H. CLARK

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EVALUATING A SATELLITE SURVEILLANCE  
SYSTEM

by

Rolf H. Clark  
Lieutenant, United States Navy  
B. S., Yale University, 1959

Submitted in partial fulfillment

for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the

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## ABSTRACT

A simple analytic model is formulated which should prove useful for evaluating a satellite surveillance system. This model describes the fraction of orbits in which a long lived satellite will "see" a selected position on the earth's surface. The fraction of orbits, denoted by  $P$ , is a function of the latitude of the ground position in question, the orbital inclination of the satellite, and the sweep width of the satellite's sensor.

$P$  is used to find estimates for the mean time to next sighting of a selected ground position given that position has suddenly become of interest, to estimate the number and locations of ground readout stations, and to estimate the number of times a satellite would sight a transiting object on the earth's surface. In addition, the application of  $P$  to the problems of optimum orbit, evasive countermeasures, and satellite weapon delivery is discussed.

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## 1. Introduction.

Rapid developments in space technology have resulted in an increased use of unmanned earth-orbiting satellites for both civilian and military purposes. These satellites provide information on outer space, the earth's atmosphere, and the earth's surface. Satellites for earth surveillance offer a distinct advantage over earth-bound systems because of their ability to scan large segments of the earth in a relatively short period of time, and because they are unencumbered by national air-space restrictions.

The Tiros and Nimbus series weather satellites represent well known examples of the use of satellites by the United States for earth surveillance. Tiros III, launched in 1961 was equipped with photographic sensors and provided information on storms and storm paths. Nimbus I, launched in 1965 served a similar purpose but made use of television cameras as sensors.

Although weather received the first attention, it is not the only phenomenon worthy of satellite surveillance. Nuclear test explosions are of sufficient interest that two Vela Hotel satellites were launched by the United States in 1965. Additional satellites apparently have been launched for certain classified military purposes. The Samos series satellites evidently have, for the last five years, been photographing military installations in Russia and Red China. [6] Seven military reconnaissance satellites (possibly of the Samos series) were reported to have been launched during 1965. [4] These satellites had polar orbits and mission lives of about five days. The observations made by these satellites are unknown, but could include ship and troop movements or test firings of ICBM's, as well as military installations.

Given a surveillance task, alternative satellite systems might be

designed to perform this task. We may want to decide which system or combination of systems is most cost-effective. Or we may want to compare the alternatives with more conventional systems such as U2 flights. To make such comparisons we must be able to evaluate the effectiveness of each type of surveillance system. This paper will be concerned with evaluating satellite surveillance systems.

What elements are important in the evaluation of a satellite surveillance system? This is a difficult question to answer. The appropriate elements must include both the nature of the phenomena to be observed, and the available hardware, including the satellite, its components, and ground support equipment. The nature of the phenomena to be observed may be such that one satellite of a particular type is sufficient. Or, it may be found that multiple satellites of one or more types are required. The appropriate number will depend upon the effectiveness of each individual satellite. Thus, an initial appraisal of a system should be based on an individual satellite's performance.

What features of an individual satellite's performance should be examined? Two features appear to be dominant. For those phenomena which occur randomly in time and location on the surface of the earth, it seems desirable to make some statement about the expected proportion of the total phenomena of interest the satellite will observe. For other phenomena we are more interested in seeing the result of an event rather than the event itself. Thus, we would like to know the average elapsed time before a satellite observes a location which has suddenly become of interest. (For example, if on the average 100 natural disasters occur in a year in the area covered by the satellite, then we may want to know the average time lapse between occurrence of a disaster and observance of the place of

disaster.)

The present procedure for satellite system evaluation is to construct an elaborate model describing the location of the satellite at all times for a given orbital inclination, altitude, etc. This approach requires that every combination of orbital inclinations, sweep width, altitude, etc., must be investigated to have a complete data bank for estimating the frequency of observation or the time before an event is observed. This, by nature of the model's complexity, requires the use of a high speed computer and much computation time. For example, if each of three parameters is to be investigated for 10 different values, then 1000 separate replications would be required.

The author believes that an alternative approach, by using a simplified analytical model, may prove to be just as useful for most appraisals of satellite surveillance systems. Appropriate analytical expressions, even if approximations to the actual situation, usually help to point out the important parameters, variables, etc. As a consequence, a simplified model like that discussed in this paper may point out some possible conclusions not immediately apparent in a computerized model. For example, it was mentioned that the United States launched seven surveillance satellites during 1965 which were characterized by polar orbits. It will be obvious from the model to be presented in this paper that for reconnaissance purposes a polar orbit is extremely inefficient unless the polar regions themselves are of particular interest.

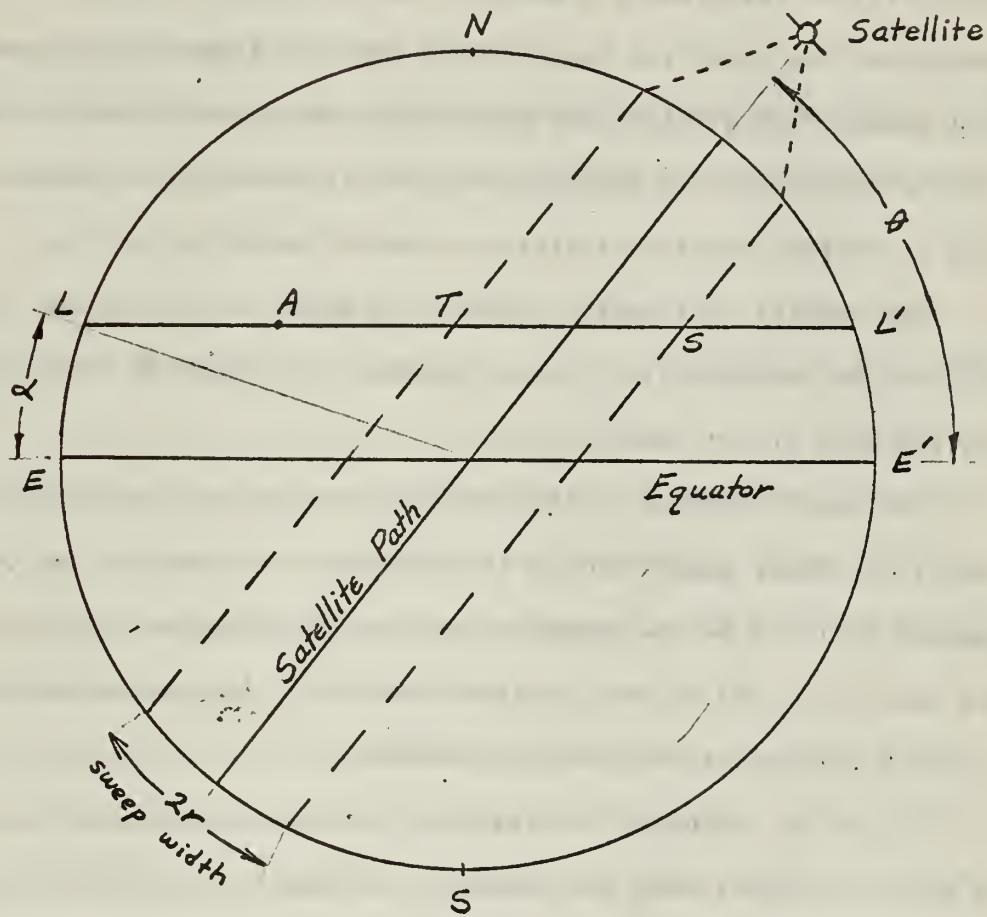
Basic to the approach is the development of a fairly simple analytical expression for the likelihood of seeing any event of interest. It would be convenient if we could find the probability that the satellite passes over a chosen ground position during any randomly selected orbit--

but since the satellite's position is completely deterministic we can not speak of the "probability" that the satellite passes over such a position. (For a selected orbit either the satellite passes over the position or it doesn't.) However it does make sense to speak of the "fraction of orbits" in which the satellite passes over a selected ground position. This distinction between "fraction" and "probability" is a fine one, but an important one. For example, if a satellite sees London every seventh orbit then it sees London on one-seventh of the orbits, but the probability it sees London on any randomly selected orbit is not one-seventh.

The likelihood function to be derived will, then, describe the fraction of orbits that the satellite spends in "positive" orbits, where a positive orbit is defined to be one such that the satellite passes over the ground position in question on that orbit. If, for example, the ground position is Beirut, then the fraction of positive orbits would be the ratio of the number of orbits in which Beirut is within the sensors sweep width to the total number of orbits (per some time interval, e.g. the life of the satellite). If  $P$  represents this fraction, then, using reasonable assumptions,  $P$  can be shown to be a function of the satellite's orbital inclination, the sweep width of the sensor, and the latitude of the point of interest.  $P$  is also a function of the satellite's altitude, in that the sweep width is usually a function of the satellite altitude.

## 2. Formulation of the Likelihood Model

The formulation of the likelihood model is facilitated by the following sketch.



Satellite motion in general is confined to being great circle motion. That is, the ground track of the satellite describes a great circle on a stationary globe. Controllable parameters are altitude, orbital inclination, and sweep width.

For surveillance purposes, upper and lower bounds exist on satellite altitude. The lower bound is caused by the earth's atmosphere. A

satellite must be outside the atmosphere to remain in orbit. The upper limit is dependent on the resolution capabilities of the satellite sensors. The lower and upper bounds are presently about 100 and 300 miles respectively. The 300 mile maximum altitude means that the time for one orbit will be less than 1.4 hours. That is, while the earth rotates one revolution the satellite has orbited about 17 times. This formulation will assume that altitude and sweep width are directly related and that knowing sweep width is equivalent to knowing satellite altitude. Therefore no further mention of altitude need be made.

The orbital inclination angle  $\theta$  is equal to the extreme latitude attained by the satellite. Once launched, we assume  $\theta$  remains unchanged for the life of the satellite.

The sweep width  $2r$  is the width of the path on the earth which the satellite sensor sweeps out as it proceeds on its orbit. For photographic sensors  $2r$  would be the width of the area photographed on one exposure, for radar it would be the length of one scan, scans being perpendicular to the direction of the satellite motion.

We let  $\alpha$  represent the latitude at which an event occurs, and since we have no control over the location (or time) of events of interest,  $\alpha$  is an uncontrolled parameter. The ground position at latitude  $\alpha$  which is of interest will be designated as A, where A is either selected randomly or non-randomly, depending on the context.

#### General Assumptions of the Model

The following general assumptions are basic to the likelihood model:

- (i) The satellite under consideration remains in orbit for long periods of time, upwards of 1000 orbits. The life of a satellite will be denoted by L.

- (ii) An event of interest is equally likely to occur at A at any time. Given an event occurs during the time L, then we assume the random variable denoting the time of occurrence is uniform on  $(0, L)$ .
- (iii)  $\theta$  cannot be varied after launch.
- (iv) The orbit is circular.
- (v) Every point on earth which lies within the sweep width of the satellite's sensor as the satellite passes overhead is seen with probability 1.
- (vi) The earth is a perfect sphere.
- (vii) The sensor points normal to the earth's surface at all times, and can see a distance  $r$  to each side of the satellite's ground track. This varies with the actual sensor aspect in designed systems (in general the sensors are aimed to the side) but simplifies the trigonometry, without changing any basic concepts of the system.
- (viii) Sensors are either photographic or radar. In either case, the ensuing model assumes perfect visibility. (Brief mention is made in Section 4 of how one might incorporate the weather problem, and consequently the problem of general atmospheric attenuation, into the model)

#### The Question of Accuracy

The formulation to be presented does not yield extreme accuracy of results. Extreme accuracy seems unnecessary in the problem of initial system evaluation. For example, if we determine, through the model, that the mean time to next sighting of a ground position A is 2 hours when in actual fact it is 2.078 hours, then this probably would be of small con-

sequence in any decision made regarding the system. Certainly it would be misleading to present results implying accurately to six or seven figures when basic assumptions preclude accuracy to more than two or three figures. Computer formulations specifically designed for the surveillance evaluation problem may claim accuracy to six figures. [3] These figures may or may not be that accurate when compared to an actual satellite in orbit, but any initial evaluation of a satellite system would probably have remained the same had the figures been rounded to two significant figures, which is approximately the accuracy inherent in the proposed model.

#### The Likelihood Function

We will denote the "fraction of positive orbits" by  $P(\theta, \alpha, r)$ . However  $P(\theta, \alpha, r)$  will often be referred to as the likelihood function  $P$ . As the notation implies,  $P$  depends on three parameters: orbital inclination  $\theta$ , latitude  $\alpha$  of the ground position A to be observed, and the sweep width  $2r$ .

The analytical expressions for  $P(\theta, \alpha, r)$  are derived in Appendix I and are given below. The derivation is based on the assumption that  $P$  is equivalent to the ratio of the sweep width measured along a latitude to one-half the circumference of the latitude. (i.e.  $\frac{TS}{LL'}$  in the previous sketch)

$$P(\theta, \alpha; r) = \begin{cases} 1 - \frac{1}{\pi} \left\{ \cos^{-1} \left[ \frac{\csc \theta}{\cos \alpha} (\sin \delta + \sin \alpha \cos \theta) \right] \right. \\ \left. + \cos^{-1} \left[ \frac{\csc \theta}{\cos \alpha} (\sin \delta - \sin \alpha \cos \theta) \right] \right\}; & 0 \leq \theta \leq \frac{\pi}{2} \\ & 0 \leq \alpha \leq \theta - \delta \end{cases}$$

$$P(\theta, \alpha, r) = \begin{cases} 1 - \frac{1}{\pi} \left\{ \cos^{-1} \left[ \frac{\csc \theta}{\cos \alpha} (\sin \delta - \sin \alpha \cos \theta) \right] \right\}; & \\ & 0 \leq \theta \leq \frac{\pi}{2} \\ & \theta - \delta \leq \alpha \leq \theta + \delta \\ 0; & \theta + \delta \leq \alpha \leq \frac{\pi}{2} \end{cases} \quad (1)$$

where  $\delta = \frac{r}{R}$  and R is the radius of the earth.

Figure 1 illustrates the behavior of P as a function of  $\alpha$  for several values of  $\theta$  when  $r = 150$  nautical miles. The figure shows that P attains its maximum when  $\alpha = \theta - \delta$ . This fact can be confirmed by an investigation of (1).

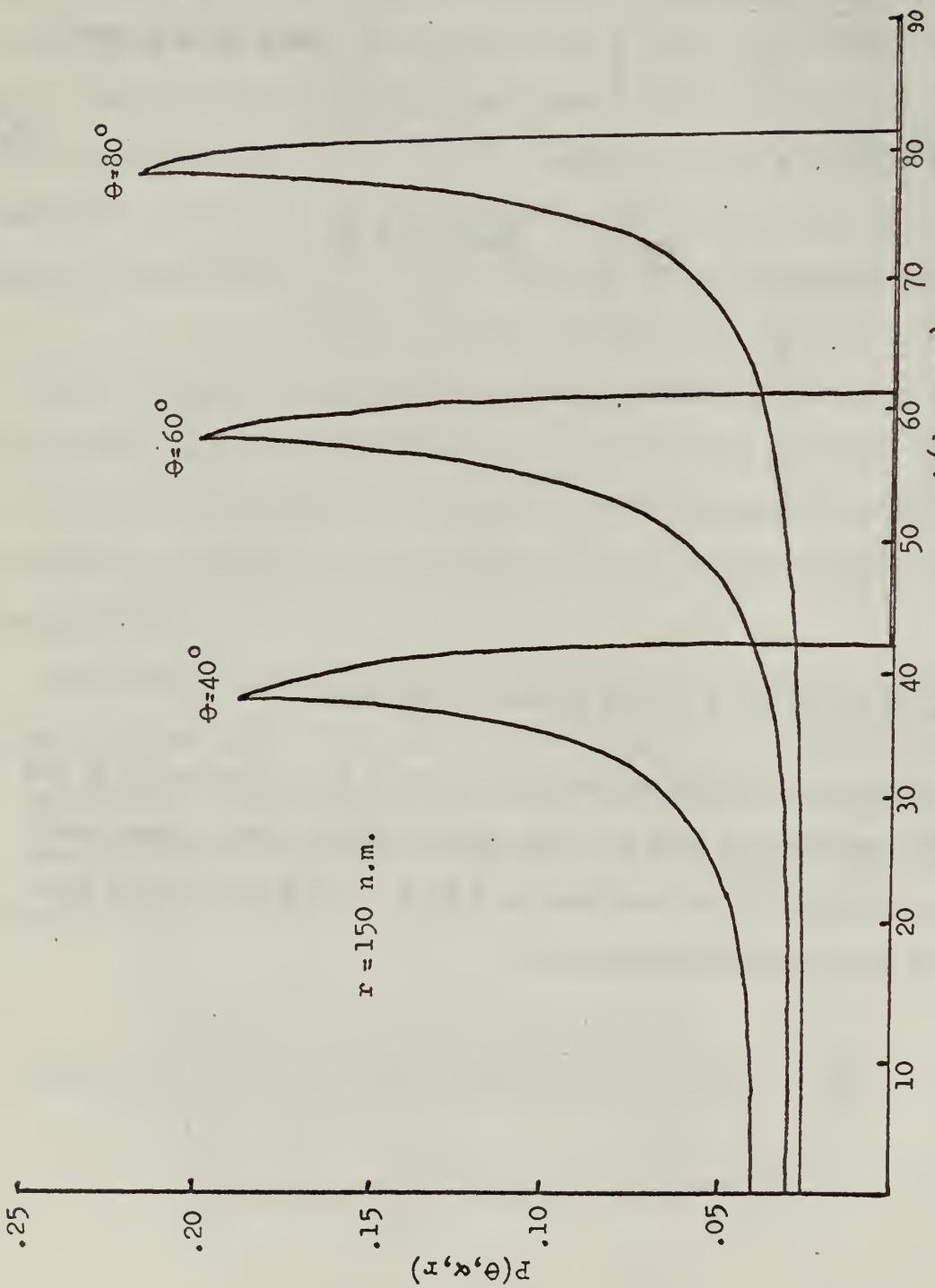


Figure 1.  $P(\theta, \alpha, r)$  as a Function of  $\alpha$  for Fixed Values of  $r$  and  $\theta$ .

### 3. Applications.

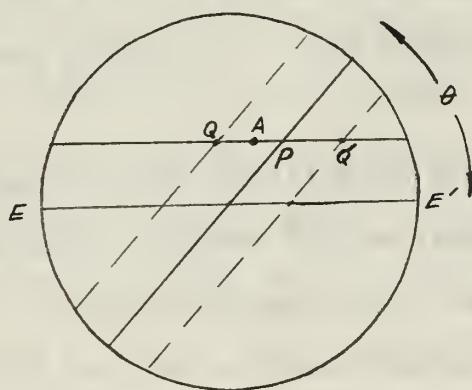
#### Mean of Time Until Next Sighting

One of the present parameters used for appraising a satellite surveillance system is the mean time between sightings.<sup>1</sup> From our model the mean time between sightings is simply  $\frac{T_0}{P}$ . However if location A has suddenly become of interest we probably would not care about the time lapse between the last sighting of A and the next one, but rather on the time lapse from the present until the next sighting. These two parameters are by no means equivalent.

In this section we will develope several estimates for the mean time from a given instant until the next sighting of a specified point A. In these developments, we will assume that phenomena of interest are equally likely to occur at A at any time during the life of the satellite.

If  $T_L$  is the time lag between occurrence of an event at location A and the first sighting of A after the occurrence, then we are looking for the mean value of  $T_L$ , denoted by  $M(T_L)$ . An exact expression for  $M(T_L)$  is difficult to obtain, but various estimates can be made.

To see why the exact expression is difficult to obtain analytically consider the following sketch.



<sup>1</sup>For an example, see [1].

If we consider an upward (downward) pass through the latitude  $\alpha$  to be one such that the satellite is moving from  $\alpha$  to a more northerly (southerly) latitude, then the satellite can see point A on any given orbit in one of two ways. Either it sees A on an upward pass or on a downward pass. The time between an upward and downward pass depends on the latitude  $\alpha$ . If  $T'$  denotes this time and  $T_o$  is the time for a satellite to make one complete orbit, then the discrete points in time at which sightings can possibly be made occur in a sequence  $T_1, T_2, \dots, T_i, \dots$ , where the times between sightings are  $T', T_o - T', T', T_o - T', \dots$

In reality, sightings do not occur in a neat sequence  $T_1, T_2, \dots, T_i, \dots$  as described above. That sequence merely tells when sightings are possible. The actual sightings may occur in sequences which are exceedingly difficult to describe analytically. For example, as indicated by the previous sketch, a sighting occurs when on an upward pass the satellite passes over point P while A lies between Q and Q'. Since the distance QQ' is on the order of several hundred miles, the speed of the earth's rotation makes it possible for A to transit QQ' in much less time than it takes the satellite to make a complete orbit. Thus the satellite may have just crossed  $\alpha$  while A was just west of Q, and A will pass Q' before the satellite makes another upward pass. A "sighting possibility" would thus be missed.

Rather than attempting the difficult task of predicting the actual sequence of positive orbits, we will develope estimates for  $M(T_L)$ . If tests show these estimates to be reasonably accurate, then they should be almost as useful to the decision maker as the true value. Let us consider three possible estimates.

Estimate I: If we assume actual sightings occur at equally spaced inter-

vals of time, then the spacing can be obtained from the likelihood function  $P$ . The assumption that events of interest occur with equal likelihood at any time during the life of the satellite implies that if an event occurs during the time  $(T_i, T_{i+1})$  then it is equally likely to occur at any instant in that interval; that is, the conditional time of occurrence  $T$  is uniform on  $(T_i, T_{i+1})$ . Since the length of this interval is  $\frac{T_o}{P}$ ,  $\hat{M}(T_L)$ , an estimate for  $M(T_L)$ , is

$$\hat{M}(T_L) = \frac{T_o}{2P}.$$

Intuition suggests that this may be a good estimate near the equator where the time between an upward and downward pass, and therefore the time between possible sightings, is the constant  $\frac{1}{2}T_o$ . Since the times of possible sightings are equally spaced, the time between actual sightings may tend to be equally spaced also. But it seems equally intiutive that as  $A$  moves away from the equator the estimate will become less reasonable.

Estimate 2: Arbitrarily define a probability model for the orbiting satellite, wherein  $P$ , instead of being the fraction of positive orbits, is defined to be the probability of a positive orbit. Further assume that all orbits are independent of each other. This model is then equivalent to one with Bernoulli trials with probability of success equal to  $P$ . Furthermore the number of trials to first success for such Bernoulli trials is a geometric random variable with mean  $\frac{1}{P}$ , where  $P$  is the probability of success on each trial. Equivalently, in terms of our model, the number of orbits to the first success (positive orbit) is  $\frac{1}{P}$  and therefore the estimate for the time to first positive orbit would be  $\frac{T_o}{P}$ .

Intuition suggests that this estimate should be good when  $\alpha$  approaches  $\theta$ , but not so good near the equator. The time between an upward and downward pass approaches zero as  $\alpha \rightarrow \theta$ , and between downward and up-

ward approaches  $T_o$ . Thus the time till possible sighting will be twice as long as that at the equator.

Estimate 3: Since neither of the above estimates seems reasonable for all values of  $\alpha$ , we return to the physics involved to see if an assumption can be made which provides a better estimate.

If a ground position A is at latitude  $\alpha$  we have said it can be seen during that orbit at one of two times, it can be seen during the upward pass or the downward pass. It is relatively easy to obtain an exact expression for  $T'$ , the time between the upward and downward passes. Knowing  $T_o$  and  $T'$  we can describe the sequence of times at which sightings are possible. We also know that the fraction of positive orbits is  $P$ .

Using these two pieces of information we assume that actual sightings occur in a sequence such that the time between sightings is proportional to the time between possible sightings, where the constant of proportionality is such that the fraction of positive orbits is  $P$ . We get (see Appendix II)

$$\hat{M}(T_L) = \frac{T_o}{\pi^2 P} \left\{ [\cos^{-1}(\cot \theta \tan \alpha)]^2 + [\pi - \cos^{-1}(\cot \theta \tan \alpha)]^2 \right\}. \quad (3)$$

Notice that for  $\alpha = 0$ ,  $\hat{M}(T_L)$  becomes  $\frac{T_o}{2P}$  which is identical to estimate 1, and for  $\alpha = \theta$ ,  $\hat{M}(T_L)$  becomes  $\frac{T_o}{P}$  which is identical to estimate 2. Thus estimate 3 varies with  $\alpha$  and also fits what intuition suggests should be the case at the extremes of the range of  $\alpha$ . Further evidence of this estimate's reasonability is given by Appendix III where  $\hat{M}(T_L)$  is compared with results obtained from a computerized simulation developed by the Naval Missile Center. [3]

### Location and Number of Readout

#### Stations Required for a Specified Level of Contact

Any study of the effectiveness of a satellite surveillance system must include an appraisal of problems of transferring the information from satellite to evaluation centers located on earth. To consider a satellite which orbits for long periods of time we should discuss readout stations. Readout stations are radio receiving stations capable of receiving signals from a satellite whenever the satellite is within a certain distance of the readout station. We will refer to the circular area wherein such reception is possible as the transmission area. As the satellite passes through a transmission area it transmits information accumulated since the last pass through a transmission area. The author knows of no present study which analyzes the optimum location of such stations or the number needed to ensure adequate transmission time, therefore it seems worthwhile to consider this question.

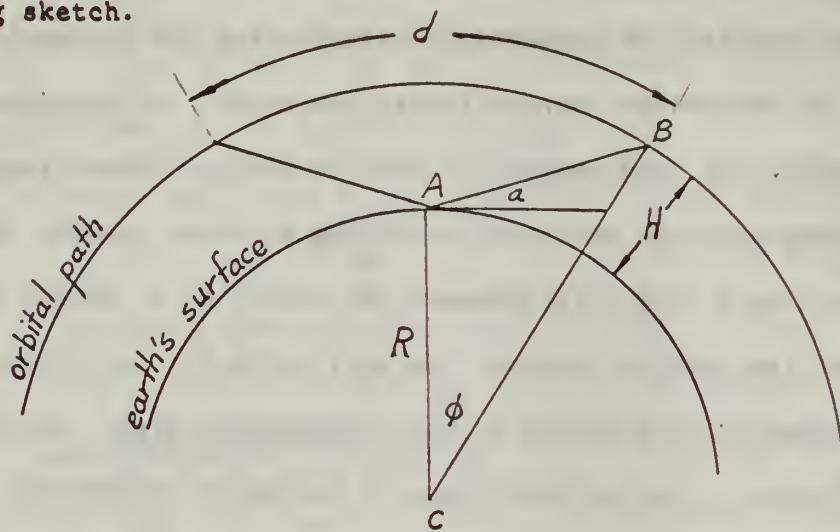
$P(\theta, \alpha, r)$  can be used to determine the percentage of time a satellite will be in contact with a particular readout station. The procedure we will use is:

1. Find the diameter of the transmission circle within which radio signals from the satellite are receivable.
2. Estimate  $M(t)$ , the mean time a satellite takes to cross the transmission circle given it intersects the circle.
3. Determine the value  $P(\theta, \alpha, r)$ , where  $\alpha$  is the latitude of the transmission circle and  $r$  is the radius of the transmission circle.
4. Estimate the mean time per orbit available for transmitting, by  $P \cdot M(t)$ .

Knowing the rate of transmission of information, a reasonable esti-

mate for the number of readout stations needed to maintain a specified level of information transmission can be obtained.

To find the diameter of the transmission circle consider the following sketch.



In this sketch  $h$  is the height of the satellite above the earth's surface, and  $a$  is the minimum reception angle above the horizon.

From the law of sines,

$$\frac{\sin(\text{angle } ABC)}{R} = \frac{\sin(a + \frac{\pi}{2})}{R+H} = \frac{\cos a}{R+H} .$$

Therefore,  $\text{Angle } ABC = \sin^{-1}\left(\frac{R \cos a}{R+H}\right)$ ,

and,  $\phi = \cos^{-1}\left(\frac{R \cos a}{R+H}\right) - a$ .

Thus, the diameter of the transmission circle,  $d$ , is

$$d = 2(R+H)\phi . \quad (4)$$

Thus we see that a readout station can receive signals from the satellite if the satellite is within a circle with diameter  $2(R + h)\phi$  with center at the readout station. If we now let  $r$  in the expression for  $P(\theta, \alpha, r)$  equal  $l(R + h)\phi$ , then we will have the expression for the fraction of orbits wherein the satellite passes within the transmission circle.

To find the mean time available for transmitting given the satellite passes through a transmission area, we will assume that the distribution of paths which the satellite makes as it crosses the transmission area is uniform; that is, the satellite is equally likely to cross the circle on any chord. Finding the mean length of path based on this assumption is not difficult. If the radius of the transmission circle is  $r$  then the mean distance travelled while in the circle is approximately  $\frac{\pi r}{2}$ . (See Appendix IV).

From the above results, we can obtain an estimate for the mean time per orbit spent in the transmission area of a single readout station. It is

$$\hat{M}(t) = \frac{\pi r}{2V} P(\theta, \alpha, r); \quad r = (R + H)$$

$V$  = ground velocity of satellite.

Because  $P$  is maximized when  $\alpha = \theta - \delta$  and  $M(t)$  is linear in  $P$ , we can maximize the mean transmission time per orbit by placing the readout station at latitude  $\theta - \delta$ .

We next consider the question of how many readout stations are required to attain at least some specified mean transmission time per orbit. We can reasonably assume that all stations would be placed close to latitude  $\theta - \delta$ . We also assume the stations are spread out on that latitude to avoid a satellite passing through two stations in rapid succession

and then not passing through another for many orbits. Under these assumptions a reasonable estimate for the number of readout stations  $N$  required to average  $t'$  minutes of transmission per orbit is

$$N = \frac{t'}{M(t)} .$$

If a value of  $t'$  is designated as the minimum mean time per orbit needed in order to receive some specified fraction of the total information acquired by the satellite, then  $N$  tells us how many readout stations are required. Such information would be needed to estimate the cost of a proposed satellite system.

#### Estimating the Number of Intersections of a Transitting Ship by a Satellite

To find an estimate for the number of intersections of a moving point on earth, consider a ship going from point  $a$  to destination  $b$ , where  $a$  and  $b$  are at latitudes  $\lambda$  and  $\beta$  respectively. Let us assume:

1. The ship proceeds at constant speed  $s$ .
2. The ship moves on great circle path.
3. The distance  $d$  from  $a$  to  $b$  on this path is known.
4. The track of the ship does not leave the area of satellite coverage.

First, to avoid complicated expressions derived from differential equations giving the instantaneous latitude of the ship as it proceeds, we elect to use a constant,  $\gamma$ , to represent the time averaged mean latitude of the ship; that is,  $\gamma$  is an estimate for the latitude which, if used to replace  $\alpha$  in  $P(\theta, \alpha, r)$ , would give the same result as if we continuously evaluated  $P$  at each instant of time during the ship's transit.  $\gamma$  is defined as

$$\gamma = \frac{\lambda + \beta}{2} .$$

Since the ship's track is a great circle track and the ship's speed is constant, this is a lower bound for the ship's time averaged mean latitude. A conservative estimate for the number of intersections will consequently be obtained.

The derivation of the estimate of the total number of sightings for a ship transiting from a to b under the above assumptions is the following:

Let  $h$  = satellite altitude

$t$  = ship's transit time from a to b

$v$  = satellite's speed

$g$  = gravitational constant.

Then

$$v = \sqrt{\frac{g}{(R + h)}} , \text{ and } t = \frac{d}{v} .$$

Now

$$T_o = \frac{2\pi(R + h)}{v}$$

and thus  $N$ , the number of orbits during the transit is

$$N = \frac{t}{T_o} .$$

If  $I$  is the number of intersections during the ship's transit then an estimate for  $I$  is

$$\hat{I} = NP(\theta, \delta, r).$$

Example: If the ship goes 1000 miles from  $10^\circ$  to  $20^\circ$  at 12 knots, the average number of sightings by a 200 mile high satellite with orbit  $\theta = 50^\circ$  and  $r = 150$  will be

$$\hat{I} = P \cdot N = NP(\theta, \delta, r) = (63.2)(.035) = 2.2.$$

#### 4. Areas for Further Study.

Having derived an expression for the fraction of positive orbits one can investigate various areas other than those already mentioned. The following are areas which seem to be of obvious interest in the evaluation task.

1. Finding the optimum orbit based on some measure of effectiveness is a problem of considerable interest. Figure 2 shows graphs indicating the distribution of major nautical activities as a function of latitude. Further work could be accomplished correlating P and these distributions to optimize the orbit. For example, if the measure of effectiveness is "number of contacts per unit time" then we could proceed as follows. Describe a graph of figure 5 by a function  $f(\alpha')$ , where  $f$  is either a discrete or continuous function approximating the graph. If we consider southern latitudes to be negative, then we can let

$$F(\alpha) = \int_{-\frac{\pi}{2}}^{\alpha} f(\alpha') d\alpha' , \text{ or } F(\alpha) = \sum_{\alpha'=-\frac{\pi}{2}}^{\alpha} f(\alpha') ,$$

depending on which is applicable. Then, let

$$G(\theta_i, r) = \int_{\alpha} P(\theta_i, \alpha, r) dF(\alpha) . \quad (10)$$

If  $\tilde{G}$  is the maximum of  $G$  with respect to  $\theta_i$ , then that  $\theta_i$  which yields  $\tilde{G}$  is the optimal orbital inclination for our chosen measure of effectiveness.

The distribution of naval activities with longitude may be combined

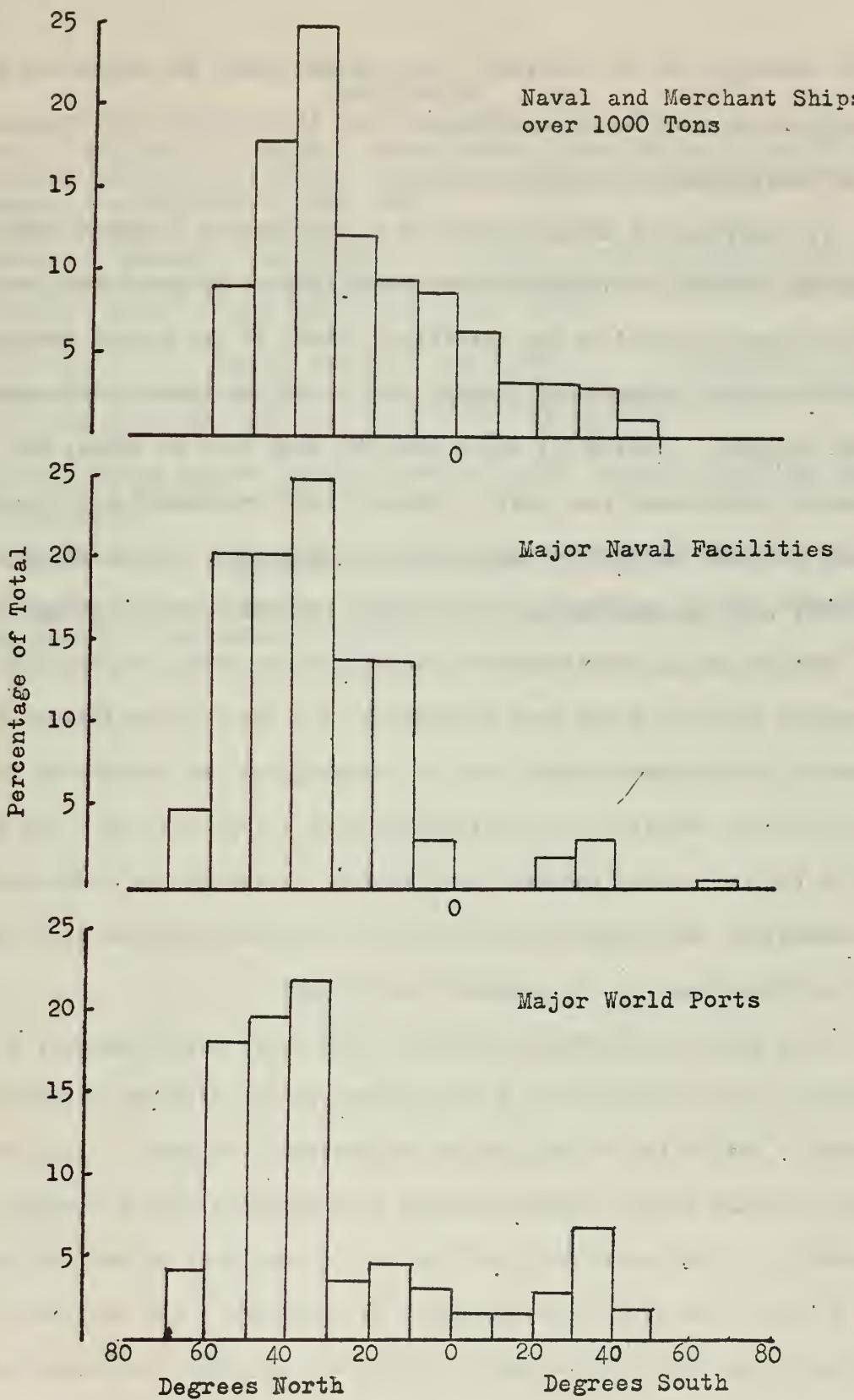


Figure 2. Distribution of Various Naval and Maritime Activities with Respect to Latitude. [2]

with the distribution for latitude. The optimum orbit for observing specified regions may then be quite different than if latitude distribution only are considered.

2. In studying the effectiveness of a satellite as a weapon delivery vehicle, we need only replace the half sweep width  $r$  by the lethal range  $r'$  of the weapon carried by the satellite. Then, if the ground position of the satellite is within the lethal range  $r'$  of the target, the weapon could be dropped.  $P$  and  $\hat{M}(T_L)$  would have the same form as above, but  $M(T_L)$  would be the mean time until a weapon could be dropped on a target after the decision to destroy the target had been made. One might correlate this with the possibility of inflight satellite course changes.

3. Investigating countermeasure techniques may prove of interest. In this respect figure 1 gives some indication of  $P$  and its sensitivity to latitude in the extreme regions near  $\theta$ . Knowing the inclination of an enemy's orbiting satellite and correlating that information with the implication of figure 1 would indicate ship routing procedures we could use to avoid detection. We obviously would try to route our ships to stay clear of the latitudes near  $\theta - \delta$ , where  $P$  is largest.

4. When data is available on global cloud cover distributions,  $P$  can be modified by the probability of cloud cover and/or darkness obscuring the sensor's vision (as in the case of photographic sensors). As a consequence,  $P$  would become closer to being a probability than a fraction. As a result of cloud cover being a function of longitude as well as latitude,  $P$  would also have to be dependent on longitude. The problem of predicting cloud cover is obviously complex and any model developed to describe the distribution of cloud cover would probably have to be a computer simulation.

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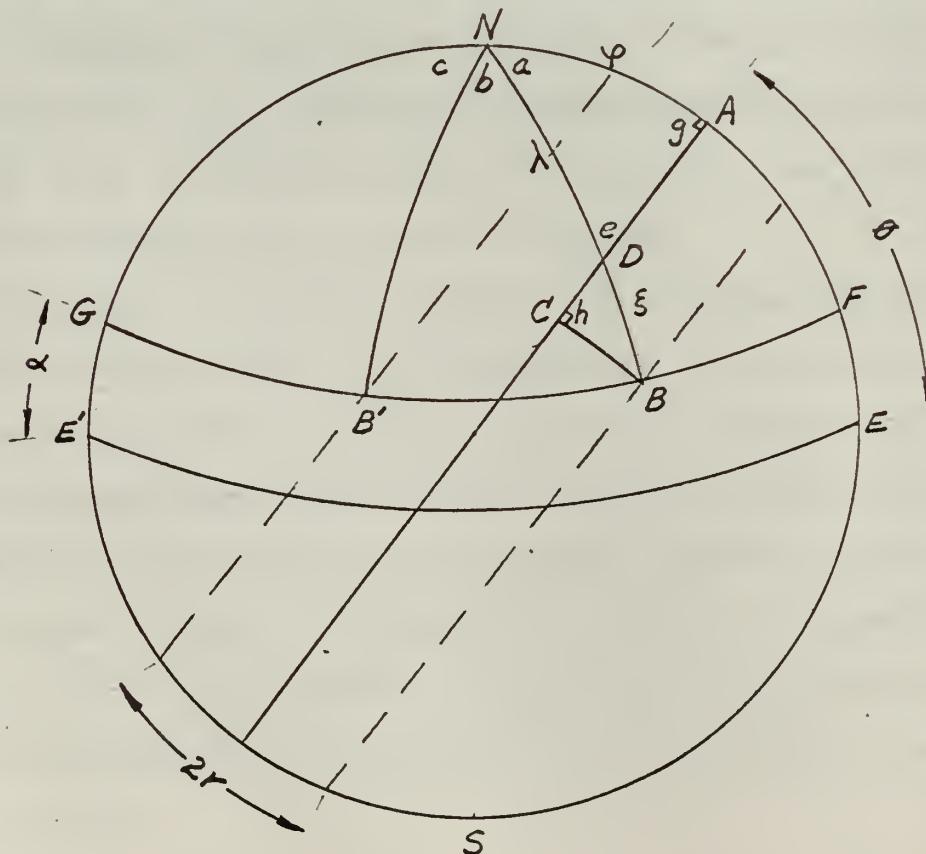
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## APPENDIX I

### DERIVATION OF THE FRACTION OF POSITIVE ORBITS

The function  $P(\theta, \alpha, r)$  is defined to be the fraction of positive orbits to total orbits given a satellite in orbital inclination  $\theta$ , a sensor sweep width  $2r$ , and a ground position at latitude  $\alpha$ . The basic rationale for finding  $P$  is to consider the circle on the sphere made by latitude  $\alpha$ , and to find the ratio of the path width of the satellite measured along latitude  $\alpha$  to one-half the total circle. Then if the ground position A is on latitude  $\alpha$ , the fraction of times A is seen will be proportional to this ratio.

To obtain an analytic expression for  $P$  based on this rationale consider the following sketch, in which capital letters denote points on the sphere, lower case letters denote angles of spherical triangles, and Greek letters denote sides of spherical triangles, a side being the usual radian measure of the central angle subtended by that side.



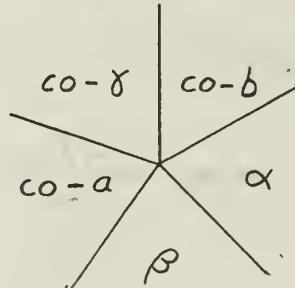
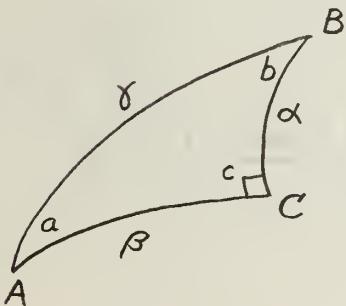
If we could determine the length of  $BB'$ , then divide  $BB'$  by  $FG$ , (one half the circumference of the circle made by latitude  $\alpha$ ) we would have  $P$ . However it is not necessary to find  $BB'$ . We get the required results by finding, instead, angles  $a$  and  $c$ , subtracting these from  $\pi$  to get angle  $b$ , then dividing  $b$  by  $\pi$ .

To find angle  $a$  we will make use of the fact that triangles  $AND$  and  $CBD$  are spherical right triangles. Furthermore, side  $\gamma$  of  $AND$  is  $\frac{\pi}{2} - \theta$ . Thus if we can find side  $\lambda$  then Napier's Rules for spherical right triangles will give us angle  $a$ . A similar approach is used to find angle  $c$ .

We first review Napier's Rules. Given a spherical right triangle as shown in the sketch below, where  $\alpha$ ,  $\beta$ , and  $\gamma$  represent the central angles subtended by the respective sides of the triangle, then the rules can be summarized as follows:

Take the five parts, excluding the right angle, and consider them to be in the circular arrangement shown. If "co-" means "complement of" then:

- (1) the sine of the middle part equals the product of the tangents of the adjacent parts;
- (2) the sine of the middle part equals the product of the cosines of the opposite parts.



To find angle  $\alpha$  we first note that  $g$  is a right angle since  $A$  is the northernmost point of the orbit, and  $h$  is a right angle by construction. The sweep width is  $2r$  and the central angle subtended by  $r$  is then

$$\delta = \frac{r}{R} .$$

From Napier's Rules,

$$\sin \delta = \cos(c - \xi) \cos(c - e)$$

and therefore

$$\sin \xi = \frac{\sin \delta}{\sin e} . \quad (11)$$

Similarly,

$$\sin \lambda = \frac{\sin \varphi}{\sin e} . \quad (12)$$

But

$$\lambda + \xi = \frac{\pi}{2} - \alpha . \quad (13)$$

From (11), (12), and (13),

$$\sin \lambda = \frac{\sin \varphi \sin \xi}{\sin \delta} = \frac{\sin \varphi \cos(\alpha + \lambda)}{\sin \delta} . \quad (14)$$

After some manipulation of (14), the expression for  $\lambda$  is

$$\lambda = \cot^{-1} \left[ \frac{\sin \delta + \cos(\xi + \lambda) \sin \varphi}{\sin(\xi + \lambda) \sin \varphi} \right] . \quad (15)$$

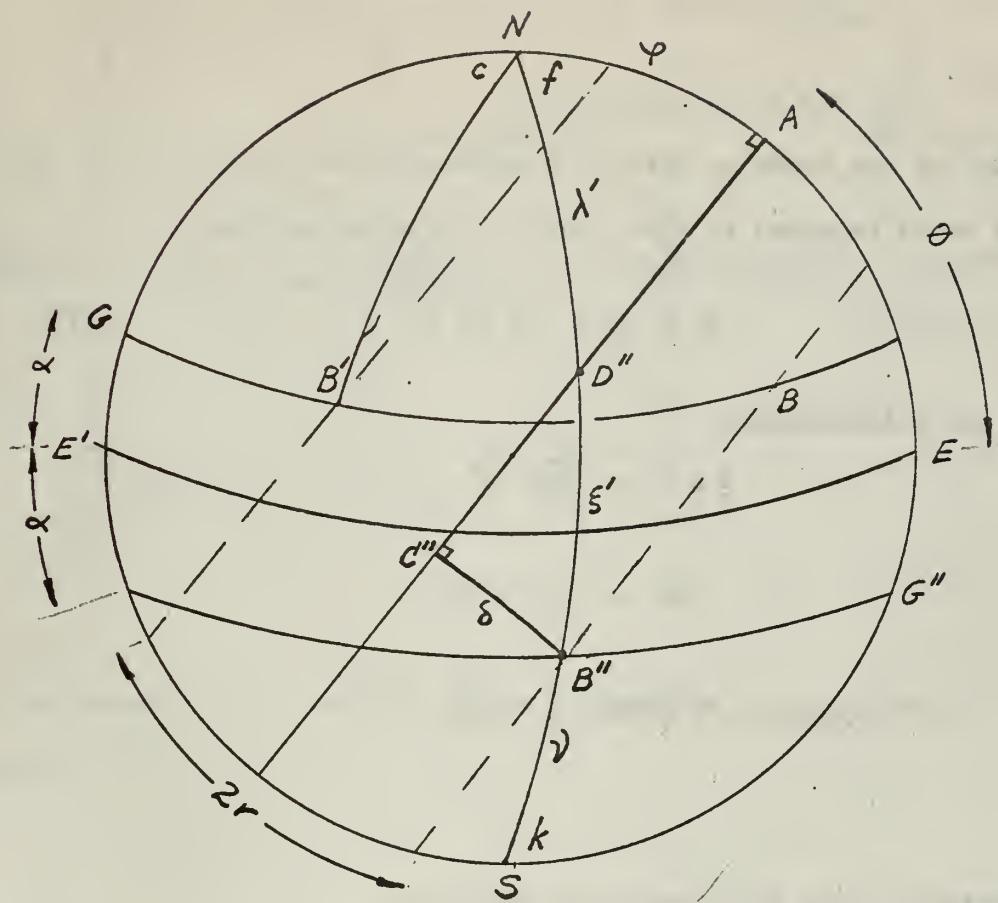
Again using Napier's Rule,

$$\cos \alpha = \tan \varphi \cot \lambda . \quad (16)$$

From (15) and (16),

$$\alpha = \cos^{-1} \left[ \sec \varphi \left[ \frac{\sin \delta + \cos(\xi + \lambda) \sin \varphi}{\sin(\xi + \lambda)} \right] \right] . \quad (17)$$

To find angle c consider the following sketch.



Due to symmetry between northern and southern hemispheres,  $B'G$  equals  $B''G''$ ; therefore angle c equals angle k. But angle k obviously equals angle f; therefore to find c we need only find f. Now f can be found in the same way we found angle a by merely using spherical right triangles AND'' and C''B''D'' instead of triangles AND and CBD. Therefore we can use equation (17) directly by replacing  $(\lambda + \xi)$  with  $(\lambda' + \xi')$ . But  $(\lambda' + \xi')$  must equal  $\pi - (\lambda + \xi)$  because  $(\xi' + \lambda')$  equals  $(\pi - \nu)$  and  $\nu$  equals  $(\lambda + \xi)$  by symmetry. Therefore, after the appropriate changes in (17), we get

$$c = \cos^{-1} \left[ \sec \varphi \left[ \frac{\sin \delta + \cos(\pi - \xi - \lambda) \sin \varphi}{\sin(\pi - \xi - \lambda)} \right] \right]. \quad (18)$$

We are now ready to obtain an expression for  $P(\theta, \alpha, r)$ . We said  $P$  would be equal to  $\frac{b}{\pi}$ , where  $b$  is obtained from

$$b = \pi - (\alpha + c). \quad (19)$$

Using the relationships

$$\xi + \lambda = \frac{\pi}{2} - \alpha,$$

$$\varphi = \frac{\pi}{2} - \theta,$$

$$\text{and } P(\theta, \alpha, r) = \frac{b}{\pi}$$

in equations (17), (18), and (19), we obtain

$$P(\theta, \alpha, r) = 1 - \frac{1}{\pi} \left\{ \cos^{-1} \left[ \frac{\csc \theta}{\cos \alpha} (\sin \delta + \sin \alpha \cos \theta) + \cos^{-1} \left[ \frac{\csc \theta}{\cos \alpha} (\sin \delta - \sin \alpha \cos \theta) \right] \right] \right\}. \quad (20)$$

On a reexamination of the derivation we realize that (20) is valid only for  $\alpha \leq \theta - \delta$ . Because angle  $\alpha$  is zero for  $\alpha > \theta - \delta$ , (20) must be modified by dropping the first term in  $\{\cdot\}$  for  $\theta - \delta \leq \alpha \leq \theta + \delta$ . Finally  $P$  must be zero for  $\alpha > \theta + \delta$ . Therefore the expression for

$P$  over the range  $0 \leq \alpha \leq \frac{\pi}{2}$  is

$$P(\theta, \alpha, r) = \begin{cases} 1 - \frac{1}{\pi} \left\{ \cos^{-1} \left[ \frac{\csc \theta}{\cos \alpha} (\sin \delta + \sin \alpha \cos \theta) \right] + \cos^{-1} \left[ \frac{\csc \theta}{\cos \alpha} (\sin \delta - \sin \alpha \cos \theta) \right] \right\}; & 0 \leq \theta \leq \frac{\pi}{2} \\ 0; & \theta + \delta \leq \alpha \leq \frac{\pi}{2} \end{cases}$$

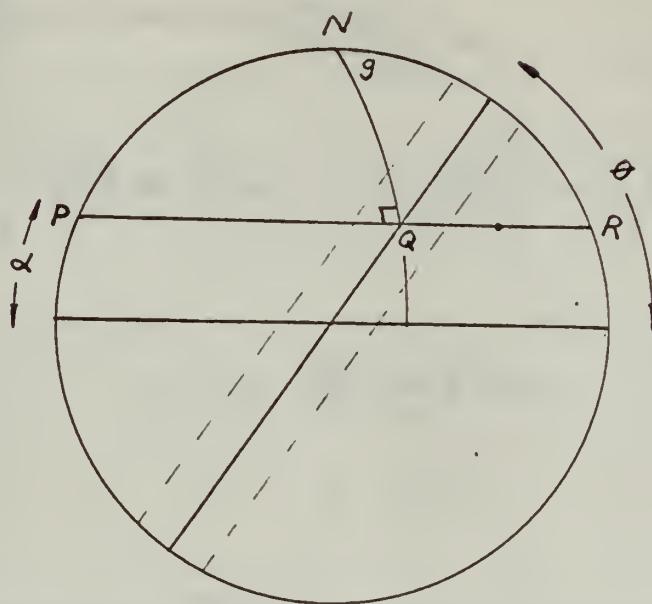
The behavior of  $P$  over the range  $0 \leq \alpha \leq \frac{\pi}{2}$  is illustrated in

figure 1.

APPENDIX II

Derivation of Estimate 3 for Mean Time to Next Sighting

Consider the following sketch:



We will consider our line of sight to be always across the plane of satellite rotation, and let the earth turn relative to that plane.

Let  $D$  = time for single revolution of earth in hours

$D'$  = time in hours for earth to turn far enough so that A (fixed on earth) moves twice the distance QR. (Thus  $D'$  is the time between a possible sighting on an upward pass, and a possible sighting on a downward pass.)

$T_o$  = Orbital period in hours

From the sketch,

$$D' = \frac{\text{arc } QR}{\text{arc } PR} D = \frac{\text{angle } g}{\pi} D.$$

But, by Napier's Rule,

$$\cos g = \cot \theta \tan \alpha .$$

$$\text{Therefore } D' = \frac{\cos^{-1}(\cot \theta \tan \alpha)}{\pi} D.$$

We want to find an estimate for the mean time to next sighting of location A, given A has suddenly become of interest. We are unable, without great difficulty, to describe the sequence of actual sighting times. Therefore we would like to define a sequence of times which we can use to approximate the sequence of actual sightings, and upon which we can base a reasonable estimate for  $M(T_L)$ .

We will assume that the sequence selected should satisfy two conditions.

Condition 1. The sequence must satisfy the condition that the mean number of orbits between positive orbits in  $n$ , where  $n = \frac{1}{P}$ .

Condition 2. The sequence should be proportional to the sequence of possible sighting times.

It is obvious why condition 1 is desired. Condition 2 is specified because it will require the sequence of assumed actual sightings to be spaced in a way which suits our intuition. Thus, if the possible sightings occur in a sequence such that the time between possible sightings has the sequence

$$1 \text{ hr}, 23 \text{ hr}, 1 \text{ hr}, 23 \text{ hr}, \dots,$$

then we would intuitively expect actual sightings to be spaced such that usually two sightings would occur relatively close together, followed by a long period with no sighting, then two more sightings relatively close together, etc.

Therefore if the times between possible sightings has the sequence

$$Z = D', D-D', D', D-D', \dots$$

then we want a constant K such that the sequence KZ satisfies our two conditions. There is obviously only one sequence which satisfies these two conditions simultaneously.<sup>1</sup> Therefore our next task is to find that sequence.

To find the sequence KZ we need only find a K which, when multiplying Z, satisfies condition 1, because any sequence KZ where K is constant will automatically satisfy condition 2. Now the sequence Z "cycles" on every second sighting possibility with a cycle time of D and therefore there are two possible sightings in a cycle of Z. We would like KZ to "cycle" on every second sighting also, but the cycle time must be equal to  $2nT_0$ , since there are to be  $2n$  orbits per every two sightings. Therefore to get KZ we must multiply Z by  $\frac{2nT_0}{D}$ . Thus:

$$K = \frac{2nT_0}{D} .$$

We then have

$$KZ = \left\{ \frac{2nT_0}{D} D', \frac{2nT_0}{D} (D-D'), \frac{2nT_0}{D} D', \frac{2nT_0}{D} (D-D'), \dots \right\}.$$

To show that this sequence satisfies condition 1 we consider a single cycle  $\left( \frac{2nT_0}{D} D', \frac{2nT_0}{D} (D-D') \right)$ . The mean time between sightings is, then,

$$\frac{1}{2} \left[ \frac{2nT_0}{D} D' + \frac{2nT_0}{D} (D-D') \right] = nT_0 ,$$

and therefore the mean number of orbits between positive orbits is n.

<sup>1</sup>We have fixed the mean time between points in the sequence and the "cycle" of the sequence.

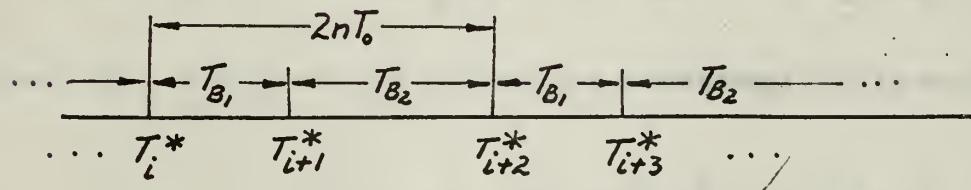
Having defined KZ, the sequence of times between assumed actual sightings, we can say that assumed positive orbits occur at times  $T_1^*$ ,  $T_2^*$ ,  $T_3^*$ , ..., where

$$T_i^* = \begin{cases} T_{i-1}^* + 2nT_0 \left( \frac{D'}{D} \right) & ; i \text{ odd} \\ T_{i-1}^* + 2nT_0 \left( \frac{D-D'}{D} \right) & ; i \text{ even} \end{cases}$$

i.e.  $\{T_1^*, T_2^*, T_3^*, T_4^*, \dots\} =$

$$\left\{ 2nT_0 \left( \frac{D'}{D} \right), 2nT_0, 2nT_0 + 2n \left( \frac{D'}{D} \right), 4nT_0, \dots \right\}$$

To estimate  $M(T_L)$  on this basis consider the following diagram:



where  $T_j^*$  indicates the time of the  $j$ th assumed sighting and the time between  $T_i^*$  and  $T_{i+2}^*$  is defined to be one cycle. Then assuming the time of occurrence of an event of interest at A is uniform over the satellite's life, we can say that the event of interest is equally likely to occur at any time  $T$  contained in  $(T_i^*, T_{i+2}^*)$ .

We are trying to estimate the mean time to next sighting given an event of interest has occurred at A. However, since each cycle is identical, we can equivalently find the mean time to next sighting given  $T \in (T_i^*, T_{i+2}^*)$ . The density of  $T$ , where  $T$  is the random variable denoting

the time of occurrence of the event of interest given  $T \in (T_i^*, T_{i+2}^*)$ ,  
is then

$$f_T(t) = \frac{1}{2nT_0} .$$

Therefore

$$\begin{aligned}\hat{M}(T_L) &= \frac{T_{B_1}}{2nT_0} \left[ \begin{array}{l} \text{mean time from } T \text{ to } T_{i+1}^* \\ \text{given } T \in (T_i^*, T_{i+1}^*) \end{array} \right] + \\ &\quad \frac{T_{B_2}}{2nT_0} \left[ \begin{array}{l} \text{mean time from } T \text{ to } T_{i+2}^* \\ \text{given } T \in (T_{i+1}^*, T_{i+2}^*) \end{array} \right] \\ &= \frac{T_{B_1}}{2nT_0} \left[ T_{i+1}^* - E(T|T \in (T_i^*, T_{i+1}^*)) \right] + \frac{T_{B_2}}{2nT_0} \left[ T_{i+2}^* - E(T|T \in (T_{i+1}^*, T_{i+2}^*)) \right] \\ &= \frac{T_{B_1}}{2nT_0} \left[ T_{i+1}^* - \frac{T_{i+1}^* + T_i^*}{2} \right] + \frac{T_{B_2}}{2nT_0} \left[ T_{i+2}^* - \frac{T_{i+2}^* + T_{i+1}^*}{2} \right].\end{aligned}$$

Substitution of the appropriate expressions for the  $T_j^*$ 's in (22) results  
in

$$\hat{M}(T_L) = \frac{nT_0}{D^2} \left[ (D')^2 + (D-D')^2 \right],$$

and, upon substitution for  $D'$  from (21),

$$\begin{aligned}\hat{M}(T_L) &= \frac{T_0}{\pi^2 P} \left[ \left[ \cos^{-1}(\cot \theta \tan \alpha) \right]^2 + \right. \\ &\quad \left. \left[ \pi - \cos^{-1}(\cot \theta \tan \alpha) \right]^2 \right]. \quad (23)\end{aligned}$$

Thus, we obtain a general estimate for  $M(T_L)$  as a function of  $\theta$ ,  $\alpha$ , and  
 $r$ .

It is of interest to note that when  $\alpha = 0$ , (23) reduces to

$$\hat{M}(T_L) = \frac{T_0}{2P} ,$$

and when  $\alpha = \theta$ , (23) reduces to

$$\hat{M}(T_L) = \frac{T_0}{P} .$$

These limiting forms are identical to estimates 1 and 2 respectively, which is a desirable result, as estimates 1 and 2 seem appealing at these limits.

### APPENDIX III

#### Comparison of Theoretical and Numerical Estimates for $M(T_L)$

Our analytical results are compared with simulated results for the following data:  $\theta = 50^\circ$ ;  $r = 150 \text{ n.m.}$ ;  $T_0 = 1.6 \text{ hrs}$ ; 105 orbits.

<u><math>\alpha</math></u>	<u>Analytical Results</u>	<u>Simulated Results</u>
0	32 hrs	35 hrs
20	22.7 hrs	22.5 hrs
40	16.5 hrs	16.7 hrs
50	12.5 hrs	10.5 hrs

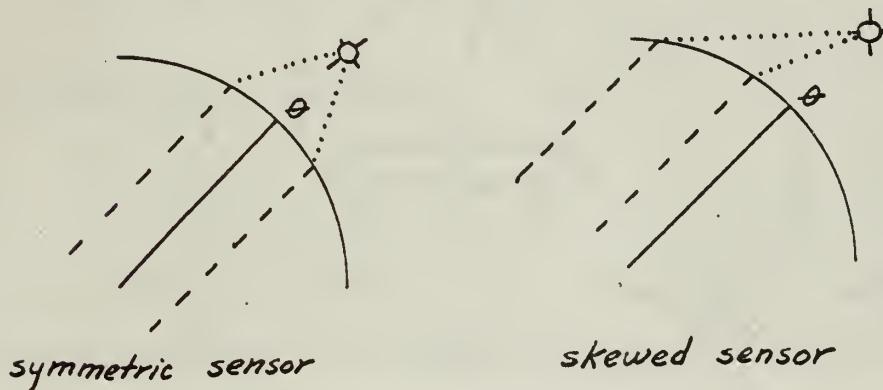
The simulated results were obtained from a Stanford Research Institute study which computed the times of intersection for each of some 1800 fixed locations. [2] To obtain  $M(T_L)$  from these times of intersection for each location the occurrence of an event was assumed to be uniformly distributed over time during the 105 orbits considered.  $M(T_j)$  for location  $j$  was then found by the formula

$$M(T_j) = \sum_{i=1}^{n_j} \frac{\alpha_i^2}{2\beta_j};$$

where  $\alpha_i; i=1,2,\dots,n_j$  are the times of intersection,  $\beta_j$  is the total time considered for location  $j$  ( $\beta_j$  = time between the first and last intersections for  $j$ ). This formula is easily verified as a formula for the time to sighting of an event which occurs uniformly in  $(0, \beta_j)$  when times of sighting occur at times  $\alpha_1, \alpha_2, \dots, \alpha_{n_j}$ . Then the overall value of  $M(T_L)$  was found by averaging the  $M(T_j)$ . Since locations were scattered throughout various latitudes only those within a small band were considered to find  $M(T_L)$  for that band. For example, all locations between  $39^\circ$  and  $41^\circ$  were used to find  $M(T_L)$  for  $40^\circ$  latitude. In the case where two cities

were so close together that considering both would create bias, (Los Angeles and Long Beach) only one was used.

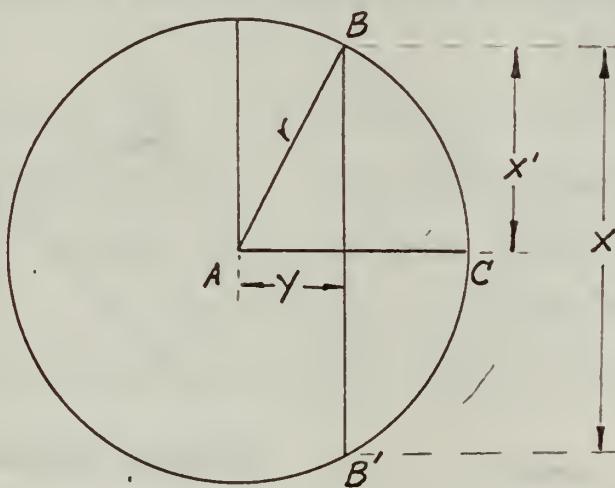
The results are extremely close with the notable exception of the range where we would expect our estimate to yield the best results, i.e. at  $\alpha = \theta = 50^\circ$ . The difference here is most likely due to the fact that the model in this paper assumes the satellite sensor "looks straight down" whereas the computer model has the sensor's field of view skewed toward the north side of the path. This difference between our model and the computer model can be seen from the following sketch.



The satellite in the latter case, when it is over latitude  $50^\circ$ , could have its sensor centered on a latitude as high as  $54^\circ$ . While a four degree error is negligible at moderate latitudes, as  $\alpha$  approaches  $\theta$  the difference becomes quite large. If the fraction of positive orbits from the computer simulation is computed and used in our formula instead of  $P(\theta, \alpha, r)$ , then  $\hat{M}(T_L)$  for  $\alpha = 50^\circ$  is found to be 10.7 hrs. Therefore the variation between our results and those of the computer simulation appears to be caused by the difference in fraction of positive orbits due to slightly different physical models.

## Derivation of Mean Distance to Cross Transmission Area

We want to find the mean distance  $X$  that a satellite travels when it transits a transmission area of a ground readout station. We can estimate this distance by assuming the transmission area is a plane circle, and that the track of the satellite is a chord through the circle with the distance from the chord to the center being uniformly distributed on  $(0, r)$ . To facilitate the discussion, consider the following sketch.



In terms of the sketch we want to find  $E(X)$ , the mean of  $X$ , when the random variable  $Y$  is uniform on  $AC$ , that is,

$$f_Y(y) = \begin{cases} \frac{1}{r} & ; 0 \leq y \leq r \\ 0 & ; \text{otherwise.} \end{cases}$$

From the sketch we get

$$X' = (r^2 - y^2)^{1/2},$$

and, because  $Y$  is uniformly distributed, the random variable  $X'$  has the distribution function:

$$\begin{aligned} F_{X'}(x') &= 1 - \Pr(Y \leq \sqrt{r^2 - (x')^2}) \\ &= 1 - \left( \frac{r^2 - (x')^2}{r^2} \right)^{1/2}; \quad 0 \leq x' \leq r. \quad (24) \end{aligned}$$

But  $X = 2X'$ ; therefore the distribution function of the random variable  $X$  is

$$F_X(x) = 1 - \left( \frac{r - (\frac{x}{2})^2}{r^2} \right)^{1/2}; \quad 0 \leq x \leq 2r. \quad (25)$$

It is then easily shown that

$$E(X) = \frac{\pi r}{2}. \quad (26)$$

Equation (26) was obtained assuming a plane circle, whereas the actual transmission area is a segment of a sphere. The result is a reasonable approximation, however, because the transmission areas are small enough that the curvature of the earth is negligible.

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## 13. ABSTRACT

A simple analytic model is formulated which should prove useful for evaluating a satellite surveillance system. This model describes the fraction of orbits in which a long lived satellite will "see" a selected position on the earth's surface. The fraction of orbits, denoted by  $P$ , is a function of the latitude of the ground position in question, the orbital inclination of the satellite, and the sweep width of the satellite's sensor.

$P$  is used to find estimates for the mean time to next sighting of a selected ground position given that position has suddenly become of interest, to estimate the number and locations of ground readout stations, and to estimate the number of times a satellite would sight a transiting object on the earth's surface. In addition, the application of  $P$  to the problems of optimum orbit, evasive countermeasures, and satellite weapon delivery is discussed.

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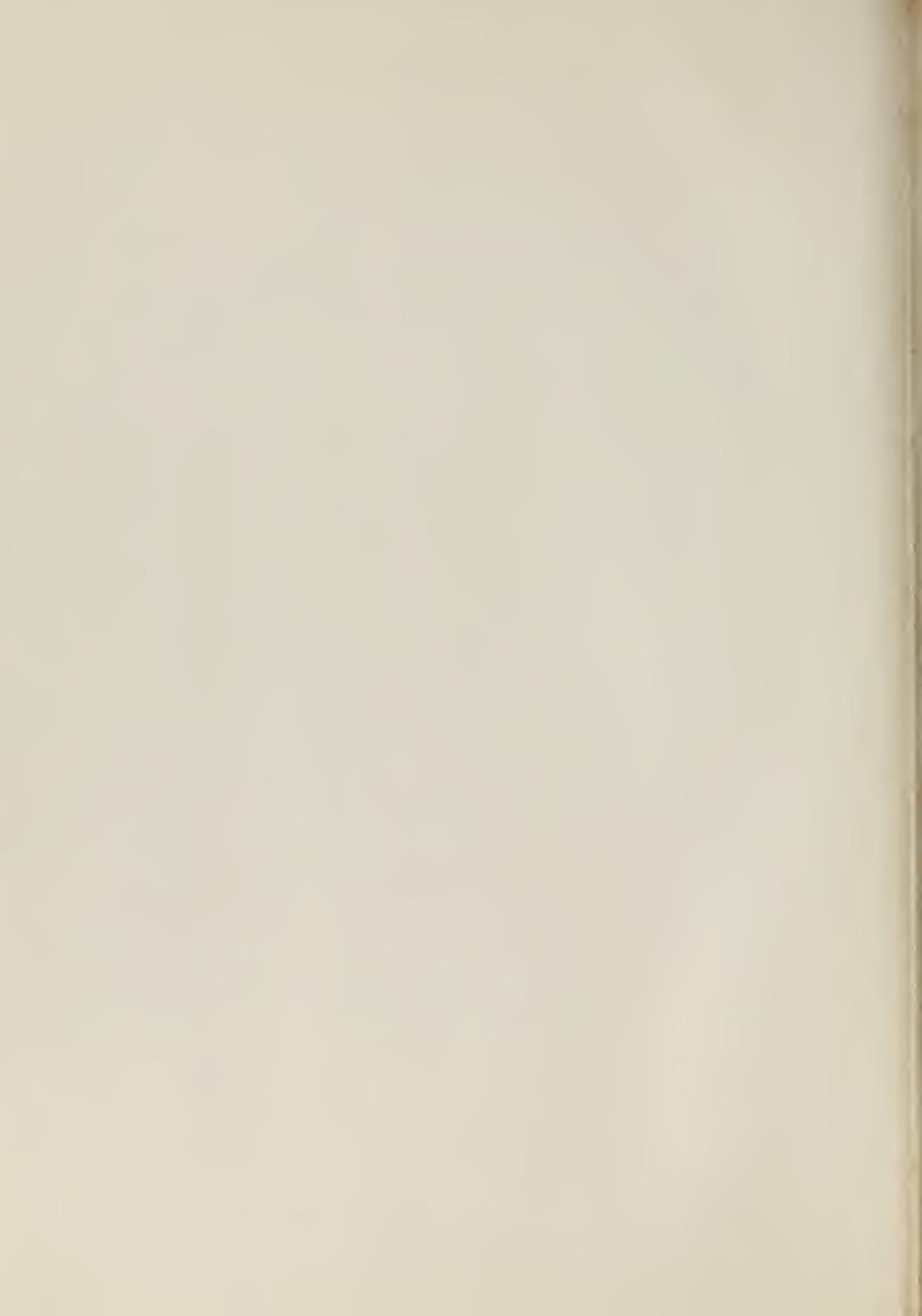
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